Analysis of a Crack Approaching a Circular Hole in Cross-Ply Laminates Under Biaxial Loading

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The problem of a crack approaching a circular hole in cross-ply laminates under uniaxial and biaxial loading is investigated in this paper. The effects of material orthotropy, geometry [R/d and a/d], and loading conditions on crack tip singularity are investigated. The stress intensity factors are obtained by the modified mapping collocation method. The present results for an isotropic infinite plate show good agreement with existing solutions. The results for cross-ply laminates show that the stress intensity factors strongly depend on material orthotropy, geometry, and loading condition. The stress intensity factors for cross-ply laminates exist between those for $\theta=0°$ and those for $\theta=90°$ in the whole range of crack length and decrease as the percentage of 0° plies increases. In the range of small crack length the stress intensity factors for biaxial tension are higher than those for uniaxial tension. In the range of large crack length the stress intensity factors for uniaxial tension are higher than those for biaxial tension.

Key Words: Modified Mapping-Collocation Method, Approaching Crack, Cross-Ply Laminates, Stress Intensity Factor, Biaxial Tension, Analytic Function, Circular Hole

1. Introduction

In Designing structures with areas of stress concentrations, cracks are very important factors for fatigue and fracture resistance of structures. Holes can be observed in many structures and are practical examples for stress concentrations. Therefore, it is desirable to investigate the problem of a crack approaching a circular hole in composite structures.

The stress intensity factors for cracks around areas of stress concentrations in isotropic plates were obtained by various techniques (Bowie, 1956; Hsu, 1975; Shivakumar and Forman, 1980; Newman, 9171; Tweed and Rooke, 1973; Tweed and Rooke, 1976). However, the solutions for cracks around areas of stress concentrations in anisotropic materials are limited due to the complexity of composite problems (Waddoups, Eisenmann and Kaminski, 1971; Wang and Yau, 1980). Recently we investigated the cracks around areas of stress concentrations in laminated composites by using a modified mapping-collocation method (Cheong and Hong, 1988; Cheong and Hong, 1989; Cheong and Kwon, 1993).

In this paper, the problem of a crack approaching a circular hole in cross-ply laminated composites is investigated. The present results for the case of is otropic infinite plate are compared with those of reference Sih (1973). Then, numerical calculations are performed for a crack approaching a circular hole in various types of cross-ply laminated composites. Uniaxial and biaxial loading conditions are also considered in this paper. Our Analysis is based on analytic function theory of complex variables. The stress intensity factors are determined from the coefficients of the stress function. The coefficients of the stress function are in turn determined from the boundary conditions by using the modified mapping-collocation method.

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2. Basic Equations of Anisotropic Elasticity

The Basic equations of anisotropic materials under plane stress condition will be considered Therefore, the stress components σ_z, τ_{yz} and τ_{xz} can be taken as zero.

When body forces are absent or are constant, the differential equations of equilibrium are

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0$$
$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0 \quad (1)$$

The equation of compatibility is

$$\frac{\partial^2\varepsilon_x}{\partial y^2}+\frac{\partial^2\varepsilon_y}{\partial x^2}-\frac{\partial^2\gamma_{xy}}{\partial x\partial y}=0 \quad (2)$$

The stress-strain relations for an anisotropic material in plane stress can be expressed as follows:

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & a_{16}\\ a_{12} & a_{22} & a_{26}\\ a_{16} & a_{26} & a_{66}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} \quad (3)$$

where a_{ij}s are compliance components.

The differential equations of equilibrium are satisfied by the introduction of a stress function $F(x, y)$, and by assuming that

$$\sigma_x=\frac{\partial^2 F}{\partial y^2},\quad \sigma_y=\frac{\partial^2 F}{\partial x^2},\quad \tau_{xy}=-\frac{\partial^2 F}{\partial x\partial y} \quad (4)$$

From Eqs. (2), (3), and (4), the general form of the stress function can be obtained as (Lekhnitskii, 1968)

$$F(x, y)=2Re[F_1(z_1)+F_2(z_2)] \quad (5)$$

where

$$z_k=x+s_k y\ (k=1,\ 2) \quad (6)$$

and F_1 and F_2 are analytic functions of the complex variables z_1 and z_2, respectively. The complex parameters s_1, s_2 are roots of the characteristic equation (Lekhnitskii, 1968)

$$a_{11}s^4-2a_{16}s^3+(2a_{12}+a_{66})s^2-2a_{26}s+a_{22}=0 \quad (7)$$

Substituting Eq. (5) into Eq. (4), the stress components are

$$\sigma_x=2Re[s_1^2\phi_1'(z_1)+s_2^2\phi_2(z_2)]$$
$$\sigma_y=2Re[\phi_1'(z_1)+\phi_2'(z_2)] \quad (8)$$
$$\tau_{xy}=-2Re[s_1\phi_1'(z_1)+s_2\phi_2'(z_2)]$$

where

$$\phi_k(z_k)=F_k'(z_k)\ (k=1,\ 2) \quad (9)$$

From Eq. (3) and the strain-displacement relations, a simple integration gives the displacement components u and v :

$$u=2Re[p_1\phi_1(z_1)+p_2\phi_2(z_2)]$$
$$v=2Re[q_1\phi_1(z_1)+q_2\phi_2(z_2)] \quad (10)$$

where p_k, $q_k (k=1,\ 2)$ are defined by

$$p_k=a_{11}s_k^2+a_{12}-a_{16}s_k$$
$$q_k=(a_{12}s_k^2+a_{22}-a_{26}s_k)/s_k\ (k=1,2) \quad (11)$$

Boundary conditions of the traction type may also be expressed as

$$f_1(s)+if_2(s)=i\int^s(X_n+iY_n)\,ds=(1+is_1)\phi_1(z_1)+(1+is_2)\phi_2(z_2)+(1+\overline{is_1})\,\overline{\phi_1(z_1)}+(1+\overline{is_2})\,\overline{\phi_2(z_2)}+c \quad (12)$$

where X_n and Y_n are the x and y components of forces exerted upon the edge per unit area. The bar notation is a conjugate symbol.

3. Theoretical Developments

We consider a crack approaching a circular hole in cross-ply laminated composites as shown in Fig. 1. The complex crack geometry and strong material orthotropy makes this problem nontrivial. The modified mapping-collocation method will be used, which was introduced in Bowie and Neal (1970), Bowie and Freese (1972).

We introduce the transformation

$$z=\omega(\zeta)=\frac{a}{2}\left(\zeta+\frac{1}{\zeta}\right) \quad (13)$$

The above mapping function maps the unit circle and its exterior in the ζ-plane onto the crack and its exterior. The other boundaries correspond to a closed contour in the ζ-plane exterior to the unit circle with co-ordinate points

$$\zeta=\frac{z}{a}+\left[\left(\frac{z}{a}\right)^2-1\right]^{1/2} \quad (14)$$

We consider now the complex variables z_1, z_2 and the additional relations :

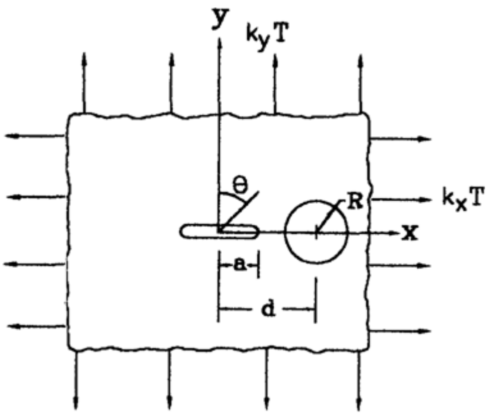

Fig. 1 Crack approaching a circular hole in cross-ply laminated composites under biaxial tension.

$$z_k=\omega(\zeta_k)=\frac{a}{2}\left(\zeta_k+\frac{1}{\zeta_k}\right)(k=1,\ 2) \quad (15)$$

Since $z=z_1=z_2$ on the crack, the parameter planes ζ, ζ_1, and ζ_2 coincide on the unit circle. Otherwise, ζ_1 and ζ_2 are distinct and are found from

$$\zeta_k=\frac{z_k}{a}+\left[\left(\frac{z_k}{a}\right)^2-1\right]^{1/2}(k=1,\ 2) \quad (16)$$

For convenience, we now define the following useful notation:

$$\begin{aligned}&\phi_k(z_k)=\phi[\omega(\zeta_k)]=\phi_k(\zeta_k),\\&\phi_k'(z_k)=\phi_k'(\zeta_k)/\omega'(\zeta_k)\,(k=1,\ 2)\end{aligned} \quad (17)$$

where

$$\omega'(\zeta_k)=\frac{a}{2}\left(1-\frac{1}{\zeta_k^{\,2}}\right)(k=1,\ 2) \quad (18)$$

From Eqs. (8) and (17), the stresses in terms of $\phi_1(\zeta_1)$ and $\phi_2(\zeta_2)$ are

$$\begin{aligned}\sigma_x&=2Re\left[s_1^{\,2}\frac{\phi_1'(\zeta_1)}{\omega'(\zeta_1)}+s_2^{\,2}\frac{\phi_2'(\zeta_2)}{\omega'(\zeta_2)}\right]\\\sigma_y&=2Re\left[\frac{\phi_1'(\zeta_1)}{\omega'(\zeta_1)}+\frac{\phi_2'(\zeta_2)}{\omega'(\zeta_2)}\right]\\\tau_{xy}&=-2Re\left[s_1\frac{\phi_1'(\zeta_1)}{\omega'(\zeta_1)}+s_2\frac{\phi_2'(\zeta_2)}{\omega'(\zeta_2)}\right]\end{aligned} \quad (19)$$

From Eqs. (10) and (17), the displacements in terms of $\phi_1(\zeta_1)$ and $\phi_2(\zeta_2)$ are

$$\begin{aligned}u&=2Re[p_1\phi_1(\zeta_1)+p_2\phi_2(\zeta_2)]\\v&=2Re[q_1\phi_1(\zeta_1)+q_2\phi_2(\zeta_2)]\end{aligned} \quad (20)$$

From Eqs. (12) and (17), the resultant-forces in terms of $\phi_1(\zeta_1)$ and $\phi_2(\zeta_2)$ are

$$\begin{aligned}f_1(s)+if_2(s)=&(1+is_1)\phi_1(\zeta_1)+(1+is_2)\phi_2\\&(\zeta_2)+(1+i\overline{s_1})\overline{\phi_1(\zeta_1)}\\&+(1+i\overline{s_2})\overline{\phi_2(\zeta_2)}+c\end{aligned} \quad (12)$$

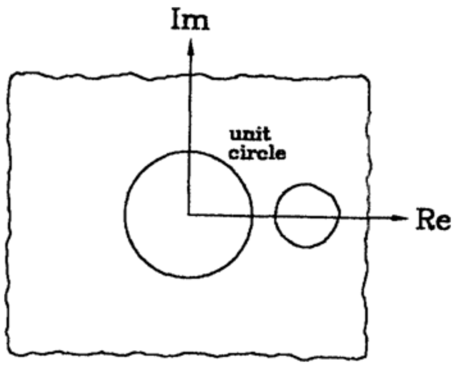

Fig. 2 ζ-transformed plane.

Let $S^+_{\zeta_1}$ and $S^+_{\zeta_2}$ denote the two parameter regions corresponding to ζ_1 and ζ_2, respectively. Their union, $S^+_{\zeta_1}$ and $S^+_{\zeta_2}$, will be denoted by S^+_{ζ}. Figure 2 shows the transformed parameter region S^+_{ζ}.

We introduce the following relation:

$$\phi(\zeta_2)=B\overline{\phi_1}\left(\frac{1}{\zeta_2}\right)+C\phi_1(\zeta_2) \quad (22)$$

where

$$\overline{\phi_1}\left(\frac{1}{\zeta}\right)=\overline{\phi_1\left(\frac{1}{\zeta}\right)} \quad (23)$$

$$\begin{aligned}B&=(\overline{s_2}-\overline{s_1})/(s_2-\overline{s_2}),\\C&=(\overline{s_2}-s_1)/(s_2-\overline{s_2})\end{aligned} \quad (24)$$

A traction-free condition on the crack can be ensured if $\phi_1(\zeta)$ is analytic in the region S^+_{ζ} and its inversion with respect to the unit circle (Bowie and Freese, 1972). If we assume that the total resultant-forces per unit thickness exerted on the hole boundary are zero, we can express ϕ_1 as follows :

$$\phi_1(\zeta)=\sum_{-\infty}^{\infty}A_n\zeta^n+\sum_{n=1}^{\infty}B_n(\zeta-r)^{-n}+\sum_{n=1}^{\infty}C_n(\zeta-1/r)^{-n} \quad (25)$$

where A_n, B_n, and C_n are complex constants and

$$r=d/a+[d/a)^2-1]^{1/2} \quad (26)$$

The problem simplifies to selecting the unknowns A_n, B_n, and C_n in Eq. (25) so that the external boundary conditions may be satisfied. The unknown A_n are directly obtained by applying the boundary conditions at infinity, given as

$$\sigma_x = k_x T, \quad \sigma_y = k_y T, \quad \tau_{xy} = 0 \tag{27}$$

Introducing the condition that the stress components σ_x, σ_y, and τ_{xy} remain bounded at infinity, and considering that the total resultant forces per unit thickness exerted on the hole boundary are zero, the structures of the stress functions for large z can be expressed as (Sih, 1973)

$$\phi_1(z_1) \to Dz_1, \quad \phi_2(z_2) \to (E+iF)z_2 \tag{28}$$

Substituting Eqs. (27) and (28) into Eq. (8), we obtain

$$\begin{aligned}
&(s_1^2+\bar{s}_1^2)D+(s_2^2+\bar{s}_2^2)E \\
&+i(s_2^2-\bar{s}_2^2)F=k_xT \\
&2D+2E=k_yT \\
&(s_1+\bar{s}_1)D+(s_2+\bar{s}_2)E \\
&+i(s_2-\bar{s}_2)F=0
\end{aligned} \tag{29}$$

Solving the simultaneous Eq. (29), we obtain

$$\begin{aligned}
D&=[k_x+(\alpha_2^2+\beta_2^2)k_y]T/\Delta \\
E&=[\alpha_1^2-\beta_1^2-2\alpha_1\alpha_2)k_y-k_x]T/\Delta \\
F&=[(\alpha_1-\alpha_2)k_x+\{\alpha_2(\alpha_1^2-\beta_1^2) \\
&\quad -\alpha_1(\alpha_2^2-\beta_2^2)\}k_y]T/\beta_2\Delta
\end{aligned} \tag{30}$$

where

$$\Delta=2[(\alpha_2-\alpha_1)^2+(\beta_2^2-\beta_1^2)] \tag{31}$$

Considering Eqs. (15) and (28), the stress functions $\phi_1(\zeta_1)$ and $\phi_2(\zeta_2)$ for large ζ_1 and ζ_2 can be expressed as

$$\begin{aligned}
&\phi_1(\zeta_1) \to (aD/2)\zeta_1 \\
&\phi_2(\zeta_2) \to [a(E+iF)/2]\zeta_2
\end{aligned} \tag{32}$$

Taking into account Eq. (22) and comparing Eq. (25) with Eq. (32), we obtain

$$\begin{aligned}
&A_{-1}=a(E-iF-D\bar{C})/2\bar{B} \\
&A_1=aD/2 \\
&A_n=0 \text{ for } n\geq 2 \text{ or } n\leq -2
\end{aligned} \tag{33}$$

Because A_0 is related to translational motion and does not affect the stress field, A_0 can be set to zero.

Thus, the stress function $\phi_1(\zeta)$ in Eq. (25) may be rewritten as

$$\begin{aligned}
\phi_1(\zeta)&=(aD/2)\zeta+(a/2\bar{B})(E-iF-D\bar{C})\frac{1}{\zeta} \\
&+\sum_{n=1}^{\infty}B_n(\zeta-r)^{-n}+\sum_{n=1}^{\infty}C_n(\zeta-1/r)^{-n}
\end{aligned} \tag{34}$$

Considering stress symmetries and cross-ply laminates, B_n and C_n are real numbers to be determined. The problem simplifies to selecting the unknowns B_n and C_n in Eq. (34) so that the boundary conditions on the circular hole may be satisfied. For purposes of numerical analysis, the terms of Eq. (34) must be truncated at $n=N$.

Substituting Eq. (34) into Eq. (19), the stresses are

$$\begin{aligned}
\sigma_x=2Re\Big[&\frac{s_1^2}{\omega'(\zeta_1)}\Big(A_1-\frac{A_{-1}}{\zeta_1^2}\Big)+\frac{s_2^2}{\omega'(\zeta_2)} \\
&\Big\{A_1\Big(C-\frac{B}{\zeta_2^2}\Big)+\Big(\overline{A_{-1}}B-\frac{A_{-1}C}{\zeta_2^2}\Big)\Big\} \\
&+\sum_{n=1}^{N}S_{1n}B_n+\sum_{n=1}^{N}S_{2n}C_n\Big], \text{ etc}
\end{aligned} \tag{35}$$

where

$$\begin{aligned}
S_{1n}&=\frac{s_1^2}{\omega'(\zeta_1)}\Big[\frac{-n}{(\zeta_1-r)^{n+1}}\Big]+\frac{s_2^2}{\omega'(\zeta_2)} \\
&\Big[\frac{nB\zeta_2^{n-1}}{(1-\zeta_2 r)^{n+1}}-\frac{nC}{(\zeta_2-r)^{n+1}}\Big] \\
S_{2n}&=\frac{s_1^2}{\omega'(\zeta_1)}\Big[\frac{-n}{(\zeta_1-1/\gamma)^{n+1}}\Big]+\frac{s_2^2}{\omega'(\zeta_2)} \\
&\Big[\frac{nBr^{n+1}\zeta_2^{n-1}}{(r-\zeta_2)^{n+1}}-\frac{nC}{(\zeta_2-1/r)^{n+1}}\Big]
\end{aligned} \tag{36}$$

Substituting Eq. (34) into Eq. (20), the displacements are

$$\begin{aligned}
u=2Re[&p_1(A_1\zeta_1+A_{-1}/\zeta_1)+p_2\{A_1(B/\zeta_2 \\
&+C\zeta_2)+\overline{A_{-1}}B\zeta_2+A_{-1}C/\zeta_2\} \\
&+\sum_{n=1}^{N}D_{1n}B_n+\sum_{n=1}^{N}D_{2n}C_n\Big], \ etc.
\end{aligned} \tag{37}$$

where

$$\begin{aligned}
D_{1n}&=p_1(\zeta_1-r)^{-n}+p_2\{B(1/\zeta_2-r)^{-n} \\
&\quad +C(\zeta_2-r)^{-n}\} \\
D_{2n}&=p_1(\zeta_1-1/r)^{-n}+p_2\{B(1/\zeta_2-1/r)^{-n} \\
&\quad +C(\zeta_2-1/r)^{-n}\}
\end{aligned} \tag{38}$$

Substituting Eq. (34) into Eq. (21), the resultant-forces are

$$\begin{aligned}
f_1=2Re[&(A_1\zeta_1+A_{-1}/\zeta_1)+\{A_1(B/\zeta_2+C\zeta_2) \\
&+\overline{A_{-1}}B\zeta_2+A_{-1}C/\zeta_2\}+\sum_{n=1}^{N}F_{1n}B_n \\
&+\sum_{n=1}^{N}F_{2n}C_n], \ etc.
\end{aligned} \tag{39}$$

where

$$F_{1n}=(\zeta_1-r)^{-n}+\{B(1/\zeta_2-r)^{-n}+C(\zeta_2-r)^{-n}\}$$
$$F_{2n}=(\zeta_1-1/r)^{-n}+\{B(1/\zeta_2-1/r)^{-n}+C(\zeta_2-1/r)^{-n}\} \quad (40)$$

Truncating the unknown terms B_n and C_n in Eq. (34) so that the boundary conditions on the circular hole are satisfied with sufficient accuracy, the stress function for a crack approaching a circular hole may be determined.

The stress intensity factors may be evaluated directly from the stress functions $\phi_1(z_1)$ or $\phi_2(z_2)$. In the limit as z_j approaches the crack tip, say z_0 ($=a$), we can express the relation between the stress intensity factors and the stress functions as follows (Sih and Liebowitz, 1968) :

$$K_I+\frac{K_{II}}{s_2}=2\sqrt{2\pi}\left[\frac{s_2-s_1}{s_2}\right]\lim_{z_1\to z_0}\sqrt{z_1-z_0}\,\phi_1'(z_1) \quad (41)$$

Considering the mapping function $z=\omega(\zeta)$ and employing Eqs. (15) ~ (18), we obtain

$$K_I+\frac{K_{II}}{s_2}=2\sqrt{2\pi/a}\left[\frac{s_2-s_1}{s_2}\right]\phi_1'(1) \quad (42)$$

Substituting Eq. (34) into Eq. (42), the stress intensity factors can be expressed in terms of coefficients of the stress function.

$$K_I+\frac{K_{II}}{s_2}=2\sqrt{\pi/a}\left[\frac{s_2-s_1}{s_2}\right]\cdot\left[A_1-A_{-1}-\sum_{n=1}^{N}\frac{n}{(1-r)^{n+1}}B_n-\sum_{n=1}^{N}\frac{n}{(1-1/r)^{n+1}}C_n\right] \quad (43)$$

Thus, we can evaluate the stress intensity factors if the coefficients of the stress functions are determined.

4. Numerical Results

The stress intensity factors for a crack approaching a circular hole in cross-ply laminates were calculated by using a computer code on the basis of the foregoing analysis. The stress intensity factors were presented as functions of the normalized crack length a/d for various types of cross-ply laminates. Considering stress symmetries, it is sufficient to apply the collocation argument over a half of the circular hole boundary in this study. Material properties of E-glass/epoxy used in the current analysis are as follows :

$E_1=53.74\text{GPa}(7.80\times10^6\text{psi})$
$E_2=17.91\text{GPa}(2.60\times10^6\text{psi})$
$G_{12}=8.96\text{GPa}(1.30\times10^6\text{psi})$
$\nu_{12}=0.25$

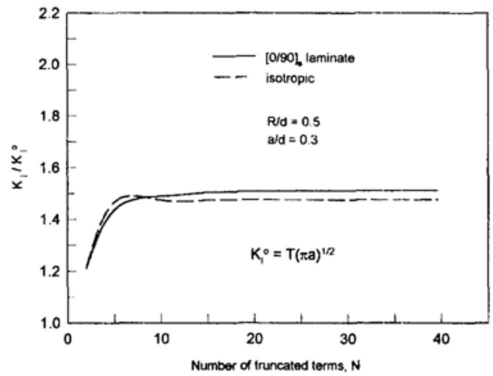

Fig. 3 Convergence curves for a crack approaching a circular hole under uniaxial tension ($k_x=0$, $k_y=1$).

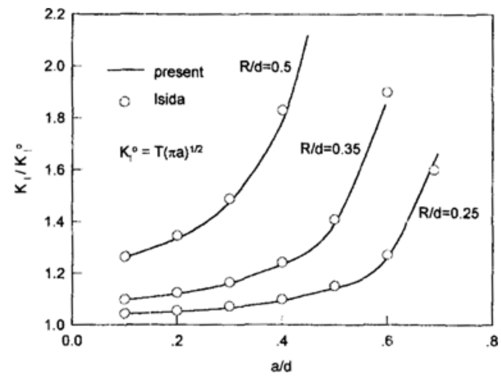

Fig. 4 Correction factors for a crack approaching a circular hole in an isotropic infinite plate under uniaxial tension ($k_x=0$, $k_y=1$).

Figure 3 shows the convergence curves for a crack approaching a circular hole in [0/90]s laminate and isotropic plate under uniaxial tension. It was found sufficient to truncate at $n=25$. In this study, stress intensity factors were normalized with those for the case of a central crack of length $2a$ in infinite plate.

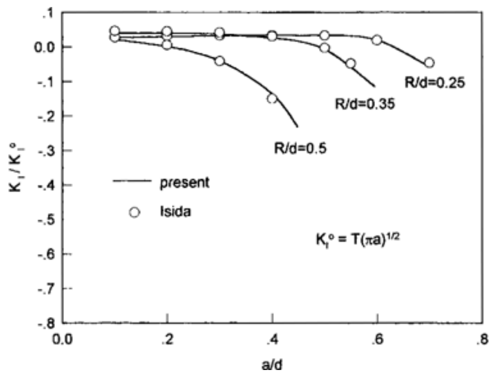

Fig. 5 correction factors for a crack approaching a ciruclar hole in an isotropic infinite plate under uniaxial tension($k_x=1$, $k_y=0$).

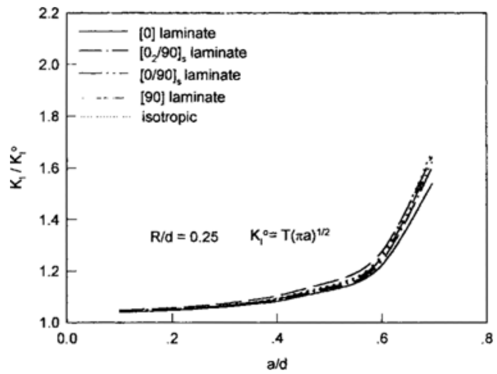

Fig. 6 Correction factors for a crack approaching a circular hole in cross-ply laminate under uniaxial tension($k_x=0$, $k_y=1$, $R/d=0.25$).

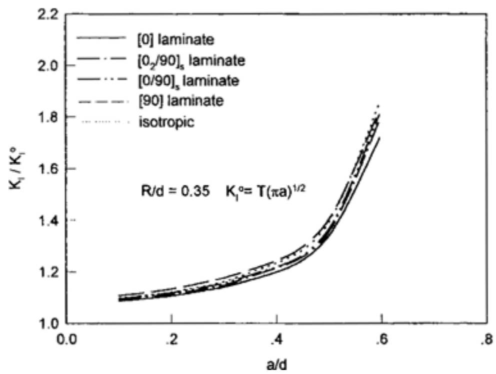

Fig. 7 Correction factors for a crack approaching a circular hole in cross-ply laminate under uniaxial tension($k_x=0$, $k_y=1$, $R/d=0.35$).

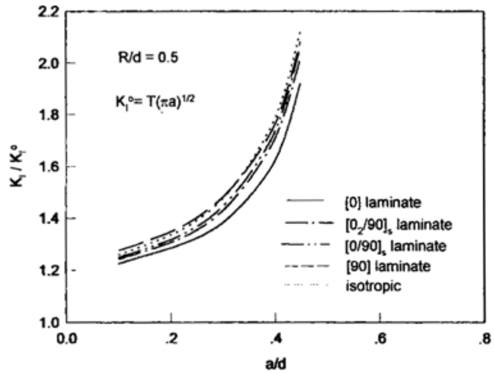

Fig. 8 Correction factors for a crack approaching a circular hole in cross-ply laminates under uniaxial tension($k_x=0$, $k_y=1$, $R/d=0.5$).

Figure 4 compares the present results for the case of isotropic infinite plate under uniaxial tension along the y-axis with those of(Sih, 1973). Figure 5 compares the present results for the case of isotropic infinite plate under uniaxial tension along the x-axis with those of(Sih, 1973). The isotropic solutions were obtained by setting the complex parameters $s_1=1.0i$ and $s_2=0.995i$. The present results almost coincide with those of (Sih, 1973).

Figures 6, 7, and 8 show the correction factors for a crack approaching a circular hole in cross-ply laminates with R/d=0.25, 0.35, and 0.5, respectively, under uniaxial tension. Each figure shows that the stress intensity factors for cross-ply laminates exist between those for $\theta=0°$ and those for $\theta=90°$ in the whole range of crack length, and decrease as the percentage of 0° plies increases. From Figs 6, 7, and 8, we can see that the difference of the stress intensity factors for cross-ply laminates becomes larger as R/d increases.

Figures 9~12 show the correction factors for a crack approaching a circular hole in [0], $[0_2/90]_s$, $[0/90]_s$, and [90] laminates with R/d=0.35 under uniaxial and biaxial tension, respectively. Figures 13~16 show the correction factors for a crack approaching a circular hole in [0], $[0_2/90]_s$, $[0/90]_s$, and [90] laminates with R/d=0.5 under uniaxial and biaxial tension, respectively. In the range of small crack length the stress inten-

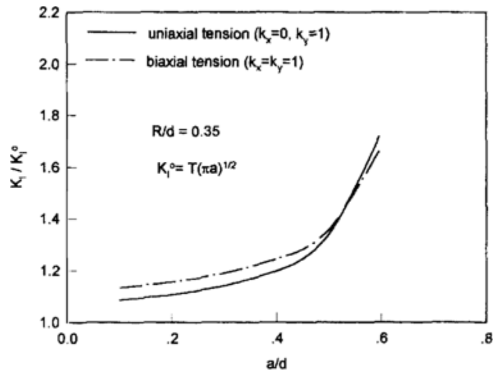

Fig. 9 Correction factors for a crack approaching a circular hole in [0] laminate under uniaxial and biaxial tensioin ($R/d=0.35$).

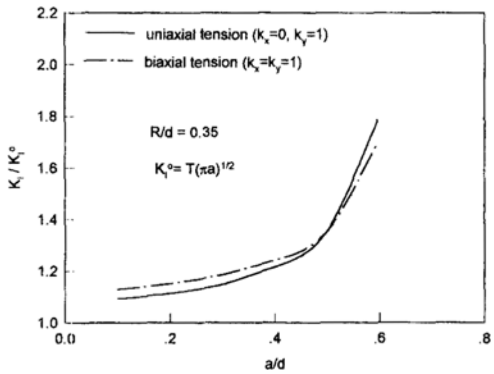

Fig. 10 Correction factors for a crack approaching a circular hole in $[0_2/90]_s$ laminate under uniaxial and biaxial tension ($R/d=0.35$).

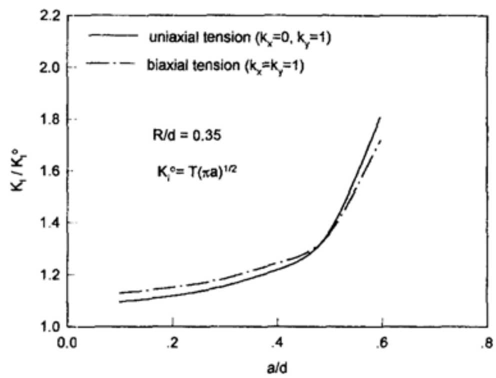

Fig. 11 Correction factors for a crack approaching a circular hole in $[0/90]_s$ laminate under uniaxial and biaxial tension ($R/d=0.35$).

sity factors for biaxial tension are higher than those for uniaxial tension. In the range of large

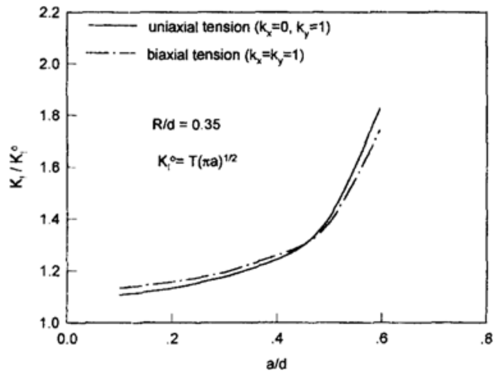

Fig. 12 Correction factors for a crack approaching a circular hole in [90] laminate under uniaxial and biaxial tension ($R/d=0.35$).

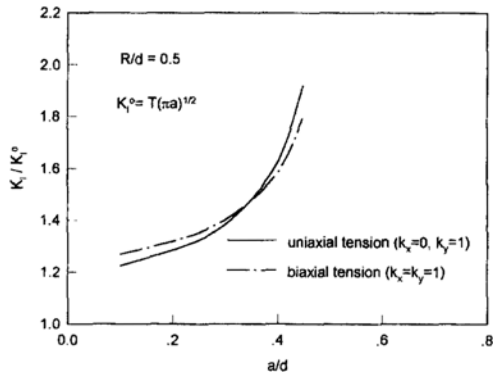

Fig. 13 Correction factors for a crack approaching a circular hole in [0] laminate under uniaxial and biaxial tension ($R/d=0.5$).

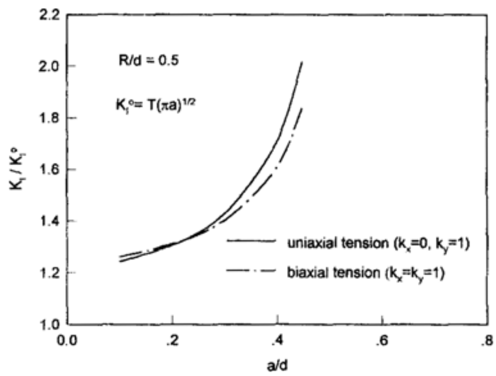

Fig. 14 Correction factors for a crack approaching a circular hole in $[0_2/90]_s$ laminate under uniaxial and biaxial tension ($R/d=0.5$).

crack length the stress intensity factors for uniaxial tension are higher than those for biaxial ten-

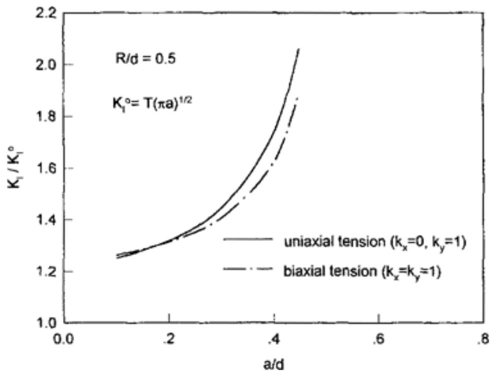

Fig. 15 Correction factors for a crack approaching a circular hole in $[0/90]_S$ laminate under uniaxial and biaxial tension($R/d=0.5$).

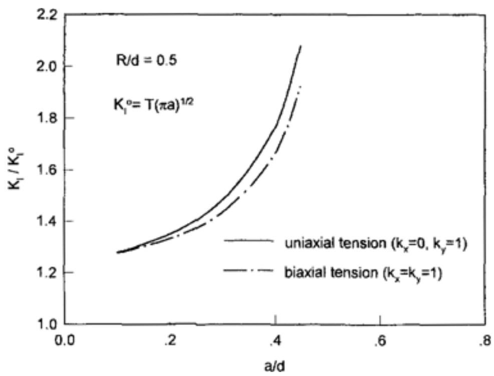

Fig. 16 Correction factors for a crack approaching a circular hole in [90] laminate under uniaxial and biaxial tension($R/d=0.5$).

sion. The more the percentage of 90° plies increases, the more rapidly the stress intensity factors for uniaxial and biaxial tension reverse. As the value of R/d becomes larger, the difference in stress intensity factors between uniaxial and biaxial tension becomes larger. As the value of R/d becomes larger, the stress intensity factors for uniaxial and biaxial tension reverse more rapidly.

5. Conclusions

Analyzing the problem of a crack approaching a circular hole in cross-ply laminates under uniaxial and biaxial tension, our conclusions can be summarized as follows:

(1) The stress intensity factors for cross-ply laminates exist between those for $\theta=0°$ and those for $\theta=90°$ in the whole range of crack length.

(3) As the number of 90° plies increases, the stress intensity factors become larger.

(4) As R/d increases, the difference in stress intensity factors for cross-ply laminates becomes larger.

(5) In the range of small crack length the stress intensity factors for biaxial tension are higher than those for uniaxial tension. In the range of large crack length the stress intensity factors for uniaxial tension are higher than those for biaxial tension.

(6) As the percentage of 90° plies increases, the stress intensity factors for uniaxial and biaxial tension reverse at smaller crack lengths.

(7) As the value of R/d becomes larger, the difference in stress intensity factors between uniaxial and biaxial tension becomes larger.

(8) As the value of R/d becomes larger, the stress intensity factors for uniaxial and biaxial tension reverse at smaller crack lengths.

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